Fast optimal control dynamics of many body states

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Abstract. We address the issue of energy measurement for accelerating the development of many-body quantum systems utilising Masuda-Nakamura's fast forward theory. We present a focus on dynamics by considering, "Is it possible to characterise, in physical terms, the exact conditions for fast forwarding? Or equivalently, for super-efficient energy measurements? What is the true physical reason for such a possibility?". So we defined super-efficient energy measurement of the quantum entanglement in the state of fast-forward dynamics. Our work shows that they can use more knowledge about the Hamiltonian and possibly quantum computational techniques such as fast forwarding or others to go beyond the Heisenberg limit.

1. Introduction

Fast-forward can be used in Bose-Hubbard or Fermi-Hubbard models to simulate quantum phase transitions that are quicker than natural. Our main results have checked cold atoms, superconducting qubits, and photonic systems. We showed a fast-forward theory and calculation for cold atoms on the optical lattice more effective than shortcuts to adiabatically (STA) [8]; in this theory, we used dynamical invariants (Lewis-Riesenfeld invariants) and modified Hamiltonian $H_{STA}(t) = H_0(t) + H_{CD}(t)$. Masuda-Nakamura's theory is optimal for modelling prolonged development in discrete increments utilizing cold atoms confined in optical lattices or Rydberg atom arrays. Counterdiabatic driving (CD) adds a control field to counteract non-adiabatic transition during cooling on cold atoms.

2. General fast-forward theory

Initially, in accordance with Ref. [1-2], we shall succinctly reiterate the explanation of the fast forward methodology for the Schrödinger equation. This section succinctly elucidates the primary elements of the theoretical framework. In the presence of the external time-dependent potential $V_0(x,t)$, the time evolution of the electronic wave function $\Psi_0(x,t)$ is governed by the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\Psi_0(x,t)}{\partial t} = H_0\Psi_0(x,t) \tag{1}$$

$$\Lambda = \bar{\alpha}t \tag{2}$$

considering a proportionality constant $\bar{\alpha}(\gg 1)$. $\Psi_0(x,\Lambda)$ means that the time evolution $\Psi_0(x,t)$ is accelerated, just like a rapid projection of a movie film on the screen. The advanced time variable can be more generalized as

$$\Lambda(t) \equiv \int_0^t \alpha(t')dt' \tag{3}$$

$$V_{FF} = V_\alpha - \frac{\alpha^2-1}{\alpha}\hbar\frac{\partial\xi}{\partial t} - \hbar\frac{\partial\alpha}{\partial t}\xi - \frac{\hbar^2}{2m}(\alpha^2-1)(\nabla\xi)^2. \tag{4}$$

We establish the initial and final conditions for the time scaling factor $\alpha(t)$ as $\alpha(0) = \alpha(T) = 1$, despite $\alpha(t) \gg 1$ in certain intervals of $0 \leq t \leq T$.Then the additional phase f vanishes at $t = 0$ and $t = 0 = T$. In this way, once we have a given electronic wave function $\Psi_0(x,t)$, we can realize its fast-forward variant $\Psi_{FF}(x,t)$ by applying the driving potential V_{FF} in Eq. (27). $\Psi_{FF}(x,t)$ completely recovers the exact fast-forwarded state at $t = T$.

3. Some examples of Fast-forward dynamics on cold atoms

In this Chapter we shall firstly show fast cooling can be done by modifying the trap frequency $\omega(t)$ in optimized way. Instead of a slow adiabatic expansion, the trap is opened rapidly with engineered path for $\omega(t)$, avoiding excitations. Our framework can be optical dipole traps use time-dependent laser intensities to archive this controlled expansion.

adiabatic dynamics, by choosing a time-dependent harmonic potential. Harmonic trap showing the expansion and contraction is described by $V_0 = \frac{1}{L^2}U_0(\frac{x}{L})$,

where L is the size of confinement. In the example of a harmonic a oscillator

$$V_0 = \frac{1}{2}m\omega^2(L)x^2 \tag{5}$$

where we use key equation for an Optimized expansion path

$$\omega(L) = \omega(0)\sqrt{\frac{\beta_0}{\beta(t)}}. \tag{6}$$

We now choose details for numeretical results $\omega(0) = 2\pi * 10^2$Hz, and $\beta_0 = 1$ initial scaling factor. Let's find out $\beta(t) = 1 + \lambda t^2$, time varies from 0 to 5 ms.

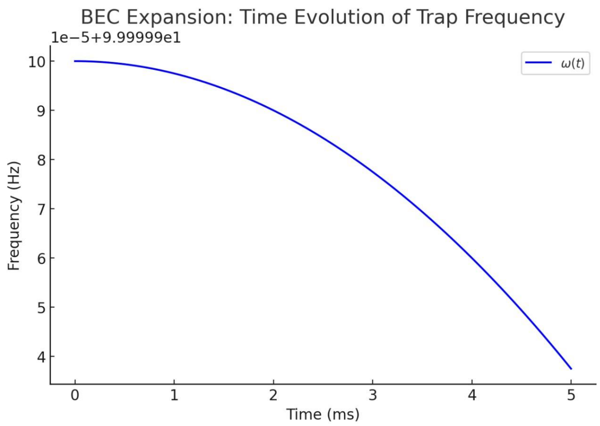

Figure 1: The plot shows how the trap frequency $\omega(L)$ decreases over time as the BEC expands after release. This follows the scaling equation, with $\omega(L)$ decreasing as $\beta(t)$ grows due to the expansion.

For a BEC in an anisotropic trap, the time-dependent width $\beta_i(t)$ (scaling factor in each direction) follows the equation:

$$\ddot{\beta(t)}_i = \frac{\omega_i^2(0)}{\beta_i \sum_j \beta_j} \tag{7}$$

where:
β_i is the scaling factor in the i th direction,
$\omega_i(0)$ is the initial trap frequency in that direction,
The product $\sum_j \beta_j$ accounts for interactions in all dimensions. For an isotropic trap, the exact solution for the frequency of the expanding cloud is:

$$\beta(t) = \sqrt{1 + \omega^2(0)t^2}$$

Thus, the exact time-dependent frequency is:

$$\omega(t) = \frac{\omega(0)}{\sqrt{1 + \omega^2(0)t^2}} \tag{8}$$

This solution describes how the trap frequency decreases over time as the BEC expands.

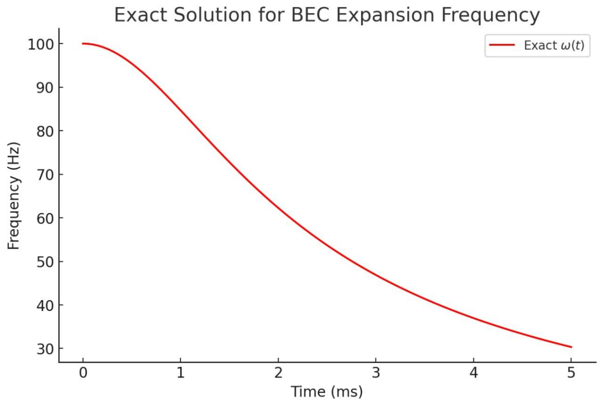

Figure 2: The plot shows the exact time-dependent frequency of the Bose-Einstein condensate during expansion. As expected $\omega(t)$ decreases over time, following the exact scaling solution:

$$\omega(t) = \frac{\omega(0)}{\sqrt{1+\omega^2(0)t^2}} \tag{10}$$

This solution is crucial for describing free expansion dynamics of a BEC in experiments.

4. Conclusion

In this study, we have explored the application of fast-forward dynamics in the context of cold atom systems, particularly focusing on Bose-Einstein condensates (BECs) in time-dependent traps. The key idea is the manipulation of the trap frequency $\omega(t)$ in a way that accelerates the evolution of the system without causing excitations, a process that differs from traditional adiabatic methods.

By introducing a time-dependent scaling factor $\alpha(t)$, we can modify the evolution of the quantum state in such a way that the system reaches its final state rapidly while maintaining the same overall properties at the initial and final times. The derived potential V_{FF}, which accounts for these modifications, ensures that the fast-forwarded state $\Psi_{FF}(x,t)$ matches the exact state $\Psi_0(x,t)$ at the boundaries, thereby allowing for accelerated but controlled dynamics.

In summary, fast-forward dynamics provides a powerful method for controlling the evolution of quantum systems, particularly in cold atom physics. The techniques explored here not only offer insight into the theoretical framework but also present practical applications for optimizing experimental protocols, such as trap manipulation and BEC cooling, while minimizing excitations. Further work in this area could include detailed experimental validation and extending these methods to more complex systems, potentially leading to more efficient and controllable quantum state manipulations in future cold atom experiments.

References

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