Radiation of ultra-relativistic electrons and positrons during channeling in crystals and in the fields of powerful lasers

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Channelling: CERN NA63-Collaboration (Aarhus, U.I.Uggerhoj el.al.

Laser: 1. Di Piazza, A., Keitel C.H., et.al Max-Planck-Institut für Kernphysik, Heidelberg, Germany

2. Russia: Kostjukov I.Yu. et.al. Institute of Applied Physics of the RAS, Nizhny Novgorod

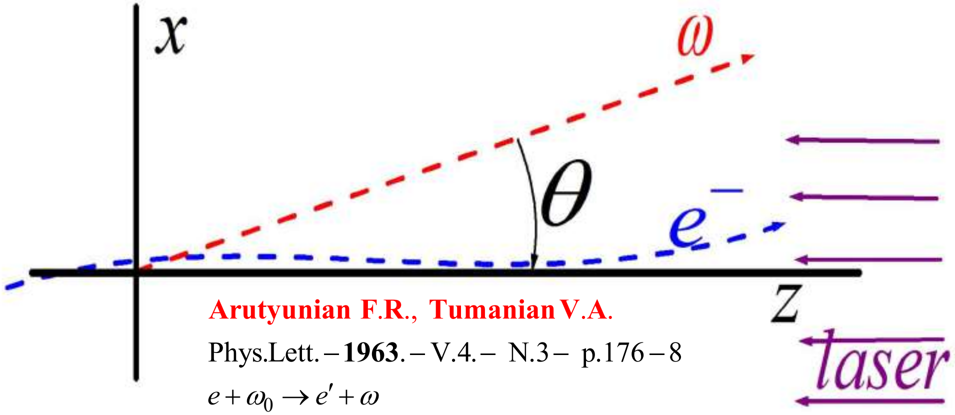
$$\mathbf{E} = -\mathbf{E}_0 \sin(\delta), \quad (1) \qquad \gamma = \gamma_0\sqrt{1+v_0^2}$$

$$\delta = \omega_0 (t + z/c),$$

$$x_m = \begin{cases} \sqrt{2}\,\dfrac{v_0 c}{\Omega_0 \gamma}, & laser \\ \dfrac{d_p}{2}\sqrt{\dfrac{E_\perp}{U_m}}, & crystal \end{cases} \quad (2)$$

$$\Omega_0 = \begin{cases} (1+\beta_0)\omega_0 \approx 2\omega_0, & laser \\ (2c/d_p)\theta_L, & crystal, \end{cases} \quad (3)$$

- Laser $\quad e + \mathrm{k}\omega_0 \rightarrow e' + \omega \quad (4)$



Arutyunian F.R., Tumanian V.A. Phys.Lett. – **1963**. – V.4. – N.3 – p.176 – 8 $e + \omega_0 \rightarrow e' + \omega$

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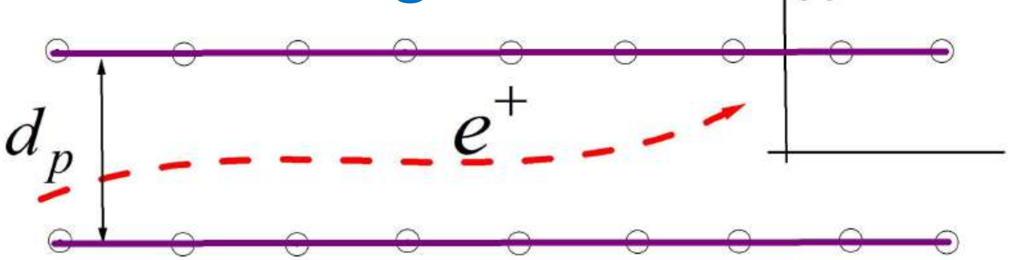
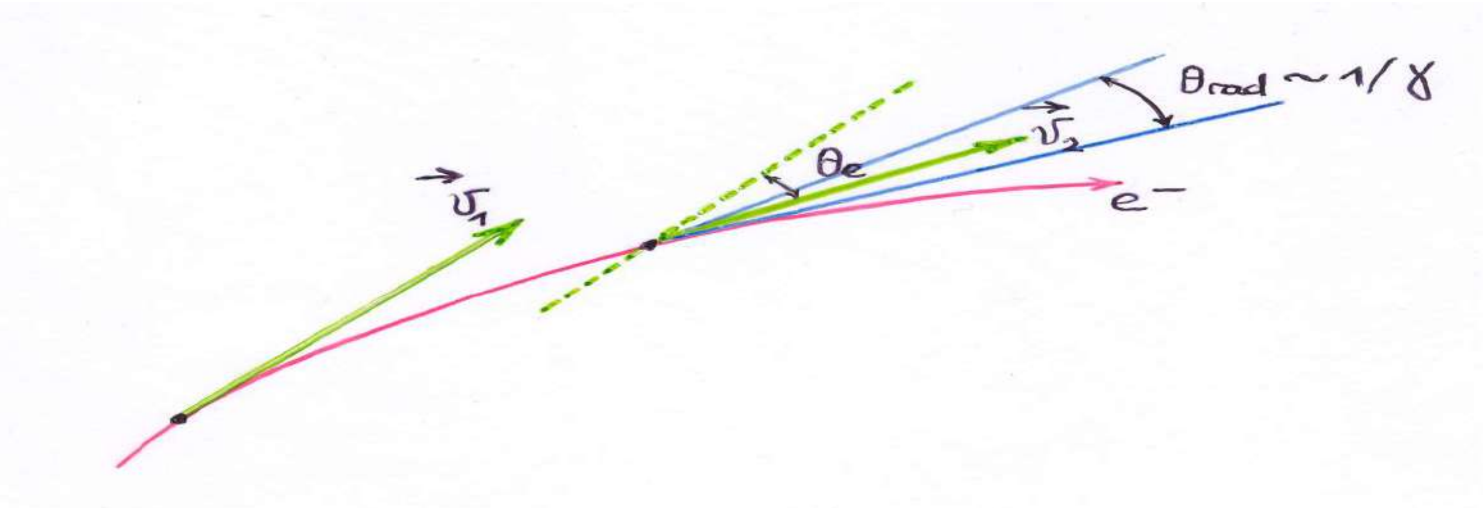
- Channeling



Radiation by relativistic electron



- **Non-dipole parameter** $D \approx \theta_e / \theta_{rad} \approx \theta_e \gamma$ (1)
- $D \gg 1$ - Synchrotron approximation $D^2 = \overline{\beta_\perp^2 \gamma^2}$ (2)
- $D \ll 1$ - Dipole approximation $\beta_\perp \gamma$ – Lorenz-invariant

Invariants

$$\nu_0^2 = \frac{e^2 \mathcal{E}_0^2}{2m^2c^2\omega_0^2} = \frac{e^2}{m^2c^4}A^2 \qquad (1)$$

$\Omega_0 = 2\omega_0$ – frequency of transverse motion

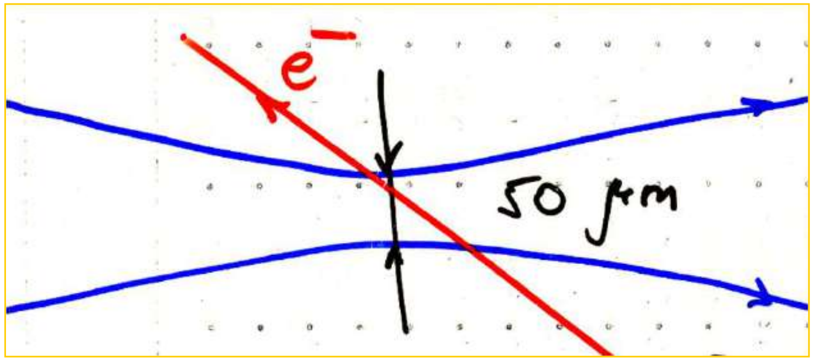
$$a = \frac{2\hbar}{m^2c^2}kp = \frac{2\hbar\Omega_0\gamma}{m\ c^2} \qquad (2)$$

$$\overline{\gamma'} = \sqrt{1+\nu_0^2}$$

in the average rest frame - ARF

$$\chi = \frac{e\hbar\left|F_{\mu\nu}p^\nu\right|}{m^2c^3} \approx \frac{2e\mathcal{E}_0\hbar}{m^2c^3}\gamma \qquad (3)$$

$$\chi \approx a\nu_0$$



Strong field effects

$$\chi = \frac{\mathcal{E}}{\mathcal{E}_c}\gamma \geq 1$$

$$\mathcal{E}_c = \frac{m^2c^3}{e\hbar} \rightarrow I \approx 10^{28}\,\frac{\mathrm{W}}{\mathrm{cm}^2}, \qquad F \approx 10^{16}\,\frac{\mathrm{eV}}{\mathrm{cm}}$$

crystals $F \approx \frac{2Ze^2}{da_F} \sim 10^3\,\frac{\mathrm{eV}}{\mathrm{A}} = 10^{11}\,\frac{\mathrm{eV}}{\mathrm{cm}}$ lab frame

$F' = \gamma F \approx 10^{16}\,\frac{\mathrm{eV}}{\mathrm{cm}}$ rest frame for $\gamma \approx 10^5$

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Comparison – lasers, crystals

Crystal (axial) :

$$E^2_{crystal} \approx K \frac{\pi Z^2 e^2}{d} N$$

520 eV/Å^3 Si <110>

580 eV/Å^3 C <100>

1000 eV/Å^3 W <111>

Laser :

10 TW,

beam radius of 50 μm

$E^2_{laser} \approx 700$ eV/Å^3

$\nu_0 = 0.3$

20 - 200 MeV electrons

For 1 TW laser, λ=1 μm

$\hbar\omega \sim 6 - 600$ keV

Channeling radiation:

$\hbar\omega \sim 6 - 200$ keV

$$D = \langle \beta_\perp^2 \gamma^2 \rangle^{1/2} = \frac{1}{\sqrt{2}} \frac{x_m \Omega_0}{c} \gamma = \begin{cases} \nu_0, & \text{для лазера,} \\ \sqrt{E_\perp E}/(mc^2), & \text{для кристалла,} \end{cases}$$

Equations of motion

$$\frac{d\mathbf{p}}{dt} = e\boldsymbol{\mathcal{E}} + e\boldsymbol{\beta} \times \mathbf{H} \quad (1)$$

$$x = c\text{æ}(\Omega_0\gamma_0)^{-1}\sin\delta$$

$$z = \beta_0 ct - c\text{æ}^2(8\Omega_0\gamma_0^2)^{-1}\sin 2\delta, \quad (2)$$

$$t = \Omega_0^{-1}\delta + \text{æ}^2(8\Omega_0)^{-1}(1-\beta_0)\sin 2\delta$$

$$\gamma_0 = (1-\beta_0^2)^{-1/2} \quad (6)$$

$$\text{æ}^2 = 2\nu_0^2/(1+\nu_0^2)$$

$$\overline{\beta_\perp^2\gamma^2} = \nu_0^2 \quad (7)$$

$$\gamma(t) = \gamma_0\sqrt{1+\nu_0^2}\left[1 + \frac{\nu_0^2}{2(1+\nu_0^2)\gamma_0^2(1+\beta_0)}\cos(2\delta)\right] \quad (3)$$

if $\gamma_0 >> 1$, and $\gamma_0 >> \nu_0$, then $\gamma = \gamma_0\sqrt{1+\nu_0^2}$ (4)

$$\delta \approx \Omega_0 t \quad (5)$$

$$x(t) = x_m \sin(\Omega_0 t),$$
$$z(t) = ct\left(1 - \frac{1}{2\gamma^2} - \frac{\langle\beta_\perp^2\rangle}{2}\right) - \frac{\Omega_0}{8c}x_m^2\sin(2\Omega_0 t), \quad (8)$$

Radiation spectrum

$$\frac{d^3N}{dud(\Omega_0 t)d\varphi} = \frac{2\alpha}{\pi}\frac{D^2}{a}\sum_{k=k_m}^{\infty}\left[g_k + \frac{uu'}{2}\left(g_k + \frac{i_z^2}{2D^2}\right)\right], \quad (1)$$

$$u = \hbar\omega / E$$

$$g_k = i_x^2 + \frac{1}{2D^2}\eta_k^2 i_z^2 - \sqrt{2}\,\frac{\eta_k}{D} i_x i_z \cos\varphi, \quad (2)$$

$$k_m = 1 + Int\left(\frac{1+D^2}{a}u'\right) \quad (6)$$

$$u' \equiv u/(1-u) \quad (7)$$

$$\eta_k = \left(\frac{ak}{u} - ak - D^2 - 1\right)^{1/2}, \quad (3)$$

$$0 < u < u_{km} = \frac{ak}{1 + D^2 + ak}, \quad (8)$$

$$i_x = \frac{1}{A}(kS_1 + S_2), \qquad i_z = \frac{1}{B}S_2, \quad (4)$$

$$S_1 = \sum_{m=-\infty}^{\infty} J_m(B)J_{k+2m}(A), \qquad A = \frac{2\sqrt{2}}{a}u'D\eta_k cos\varphi,$$

$$S_2 = \sum_{m=-\infty}^{\infty} mJ_m(B)J_{k+2m}(A), \qquad B = \frac{u'}{2a}D^2 \quad (5)$$

1. Dipole approximation $D \ll 1$

$$\frac{1}{n_0}\frac{dN}{du} = \frac{3}{2a}\left[1 - \frac{2u}{a(1-u)} + \frac{2u^2}{a^2(1-u)^2} + \frac{u^2}{2(1-u)}\right],\ u \le u_m \quad (1)$$

$$u_m = \frac{\omega_m}{E} = \frac{a}{1+a} \quad (2)$$

$$\frac{dN}{d(\Omega_0 t)} = \frac{2}{3}\alpha D^2 \equiv n_0 \quad (3)$$

2. Constant field approximation - CFA

$$\frac{D^2}{a} > 1, \quad \left(\text{or} \quad \frac{D^2}{a}\frac{u}{1-u} > 1\right) - \text{CFA} \quad (4)$$

$$\frac{d^2N(t)}{dud(\Omega_0 t)} = \frac{2}{3}\alpha\frac{\sqrt{3}}{\pi a}J_{SYN}(\xi) \quad (5)$$

$$\xi = \frac{2}{3\chi}\frac{u}{1-u} \quad (6)$$

$$\chi = aD\sin\alpha_0 / \sqrt{2} \quad (7)$$

$$J_{SYN}(\xi) = \left(2 + \frac{u^2}{1-u}\right)K_{2/3}(\xi) - \int_{\xi}^{\infty} K_{1/3}(x)dx = \quad (8)$$

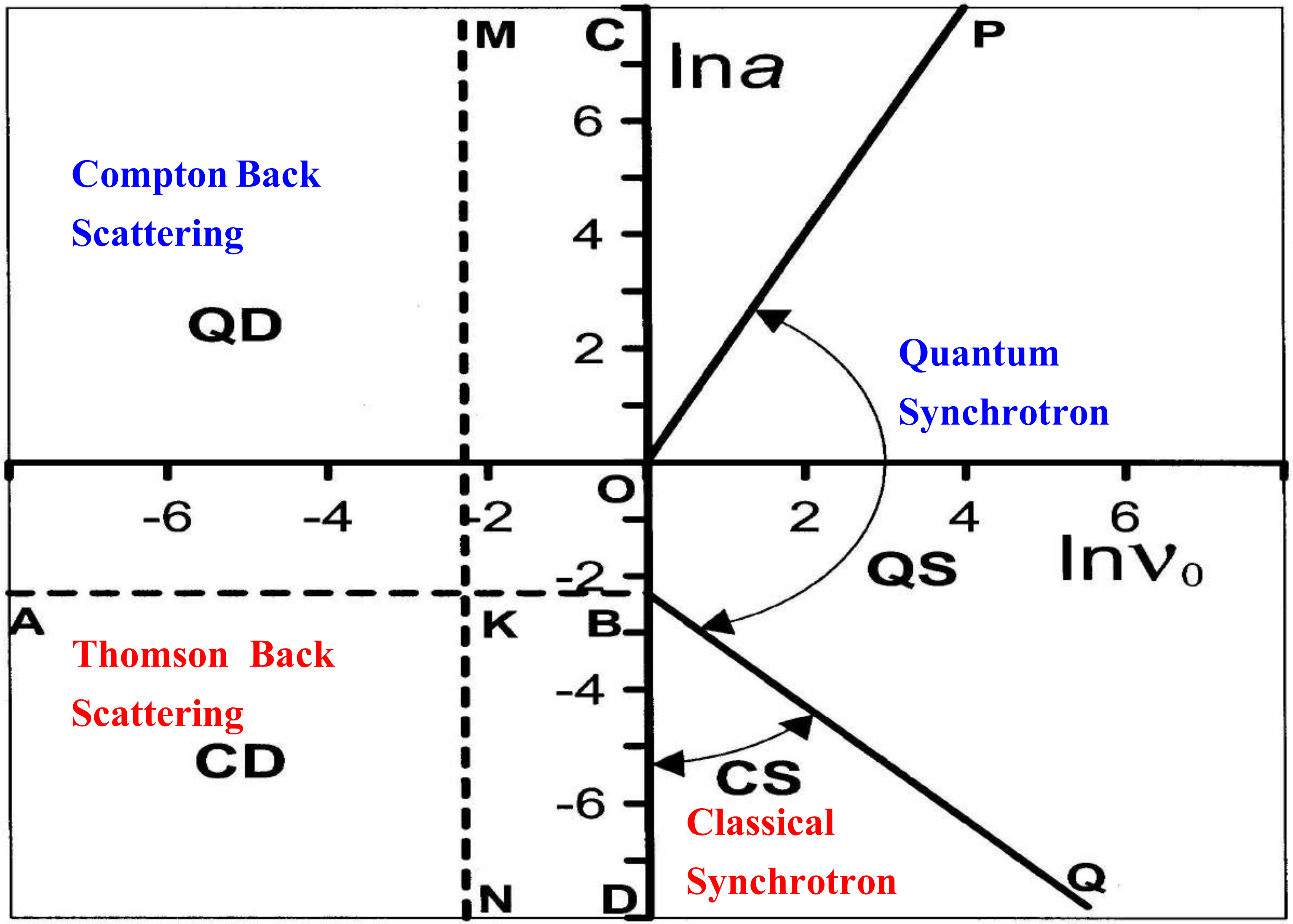
$$= \sqrt{3}\int_0^{\infty}\left[\frac{3 + 12x^2 + (16/3)x^4}{3 + 4x^2} + \frac{u^2}{1-u}\left(1 + \frac{2}{3}x^2\right)\right]\frac{\exp\left[-\xi\left(1 + 4x^2/3\right)\sqrt{1 + x^2/3}\right]}{\sqrt{1 + x^2/3}}dx \quad (9)$$

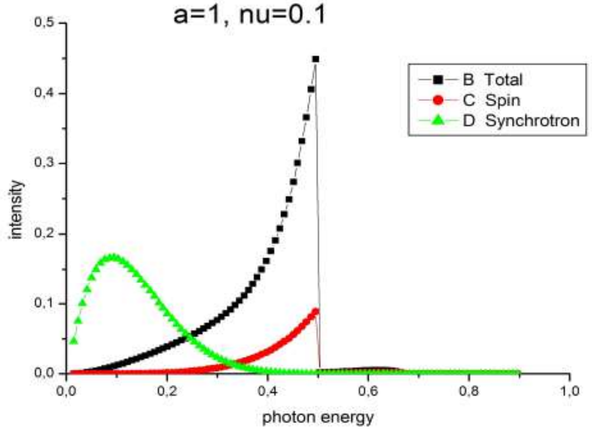
M.Kh.Khokonov. JETP, V.99, No.4, pp. 690-707, (2004); Physica Scripta V.55, No.5, P.513-519, (1997)

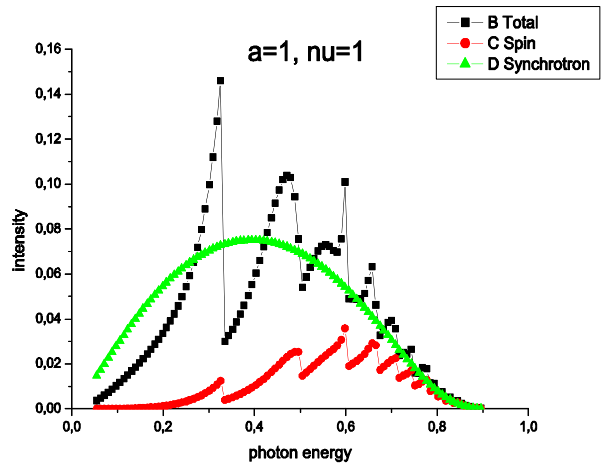
General Classification

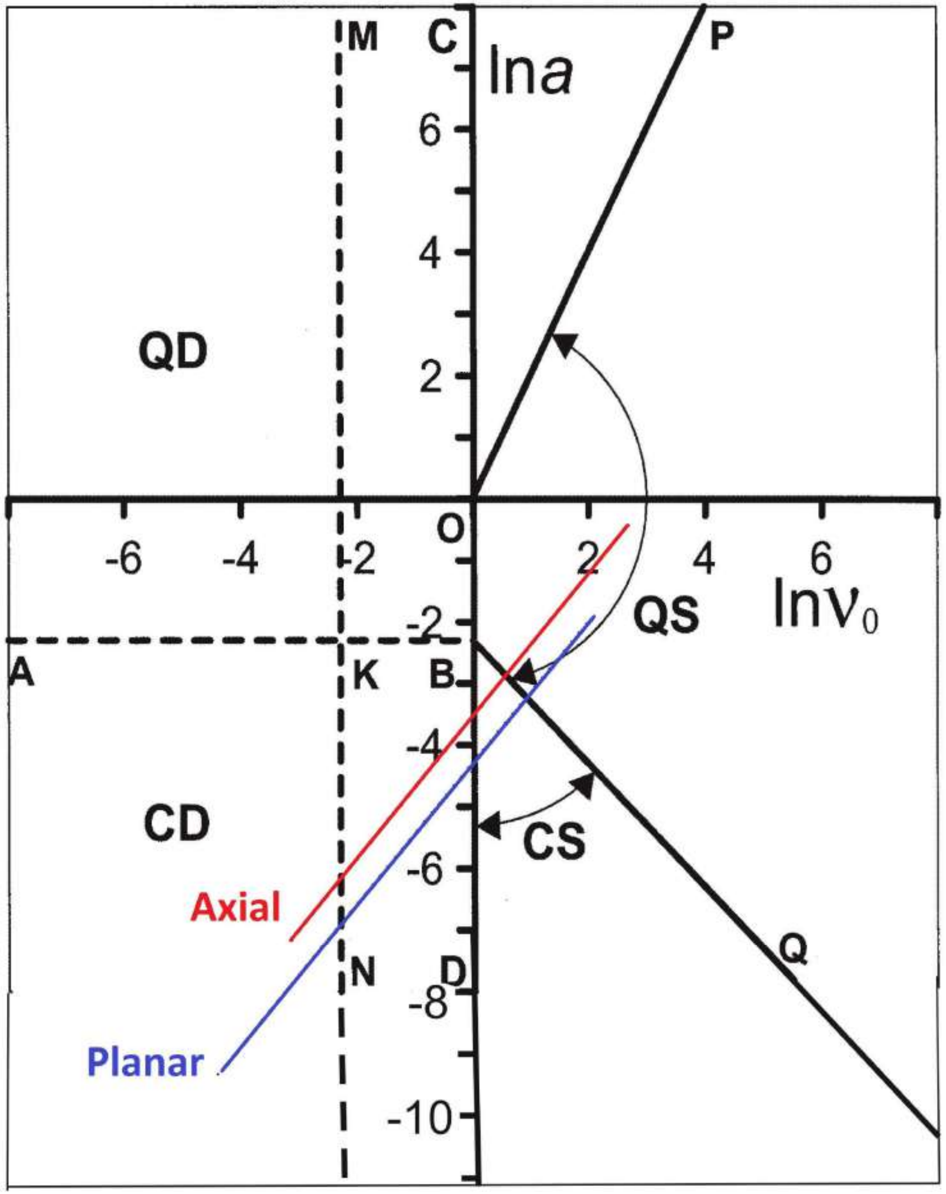
$E = 125$ GeV

$\lambda = 1$ μm









Channeling

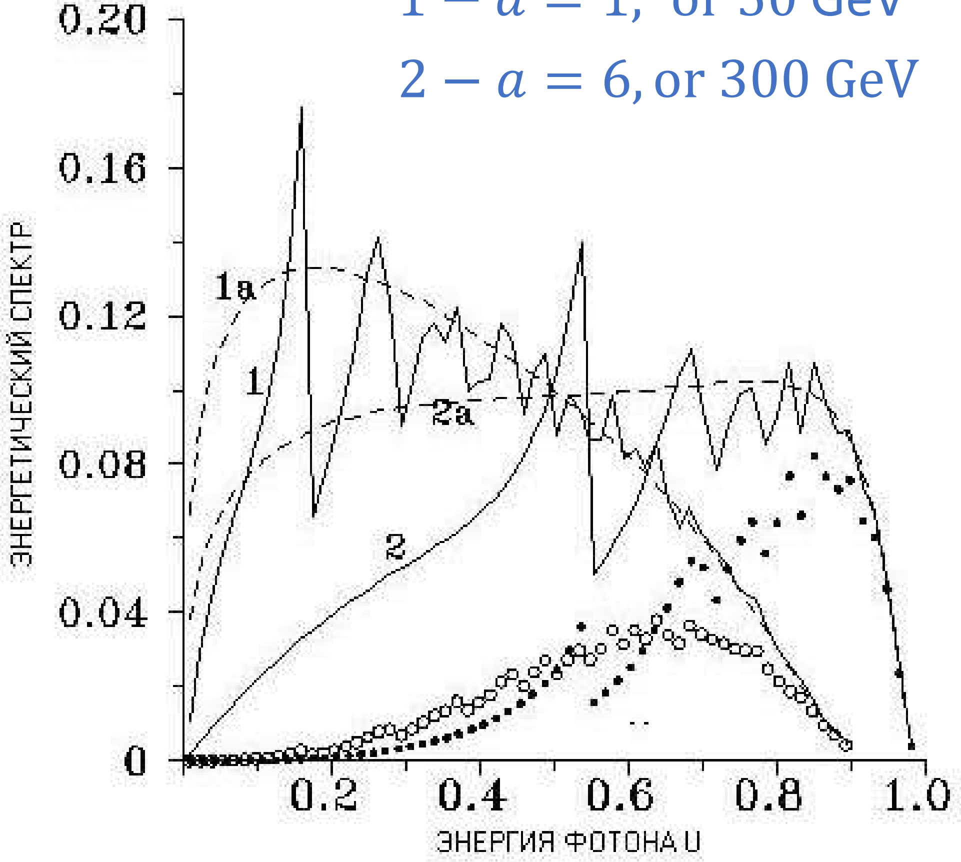
$$D = a\frac{mc^2 d_p}{4\hbar c}\sqrt{\frac{E_\perp}{2U_m}} \qquad (1)$$

$$a \approx \frac{2\hbar c}{mc^2} \times \begin{cases} \frac{1}{a_F} D, & axial \\ \frac{1}{d_p} D, & planar \end{cases} \qquad (2)$$

$D = \nu_0 = 2$

$1 - a = 1$, or 50 GeV

$2 - a = 6$, or 300 GeV



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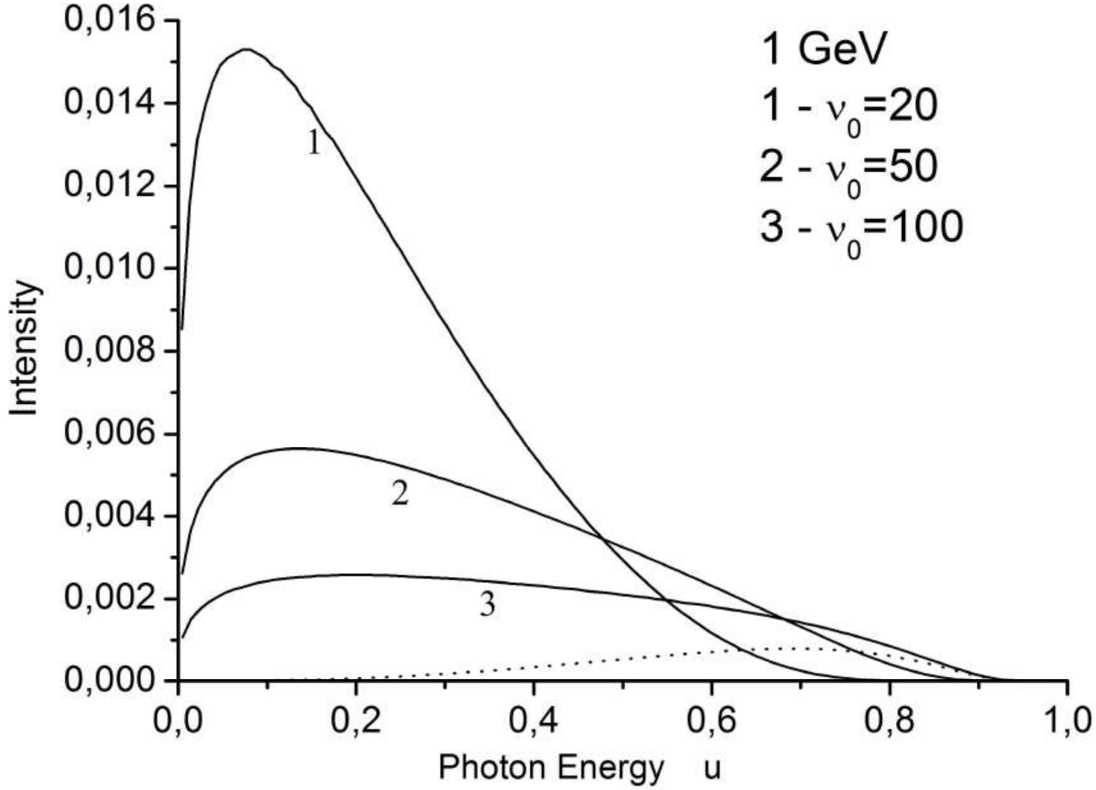
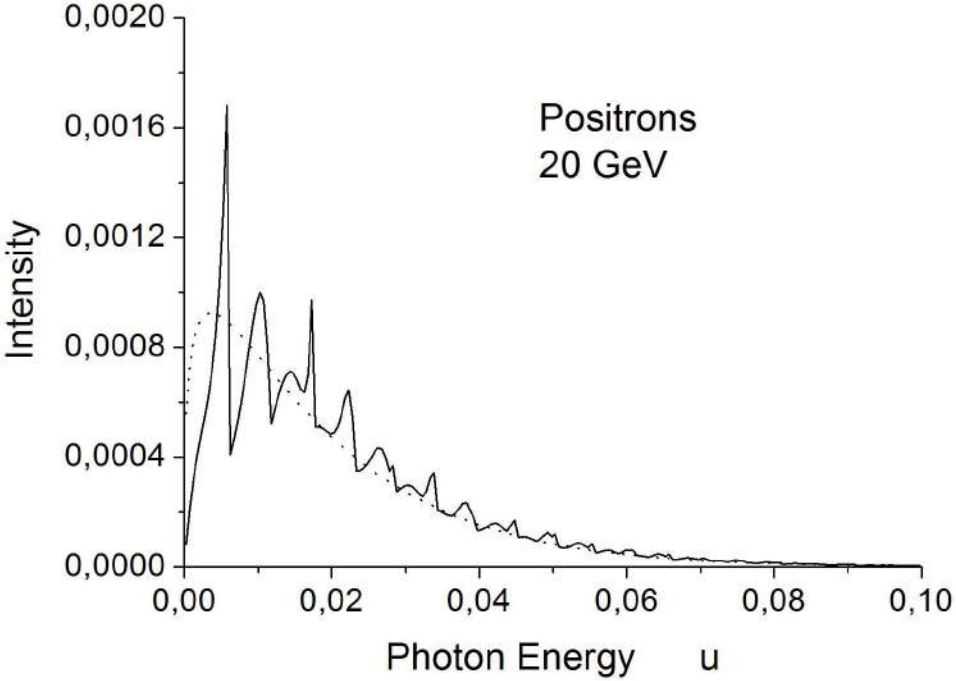
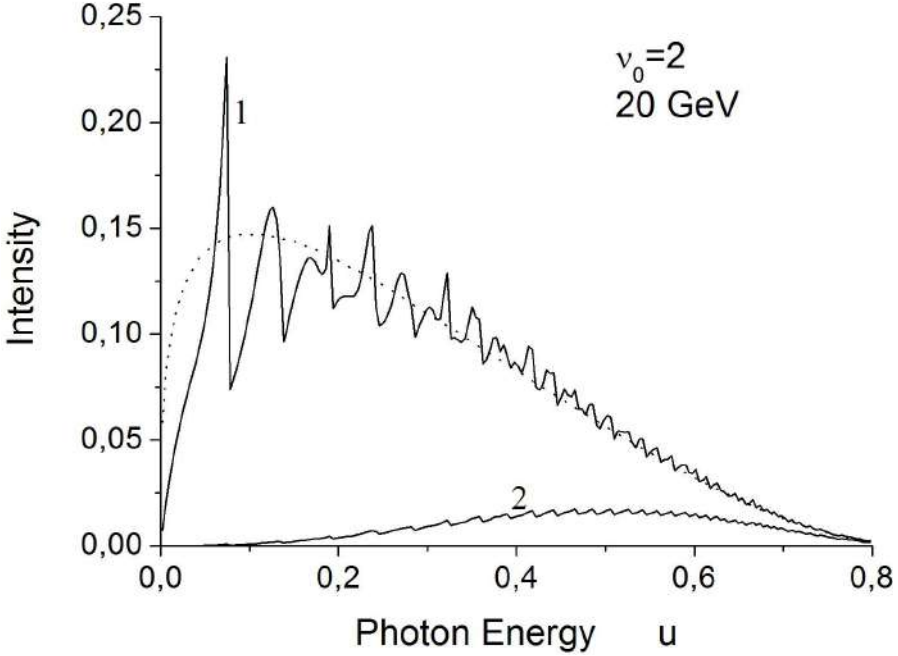
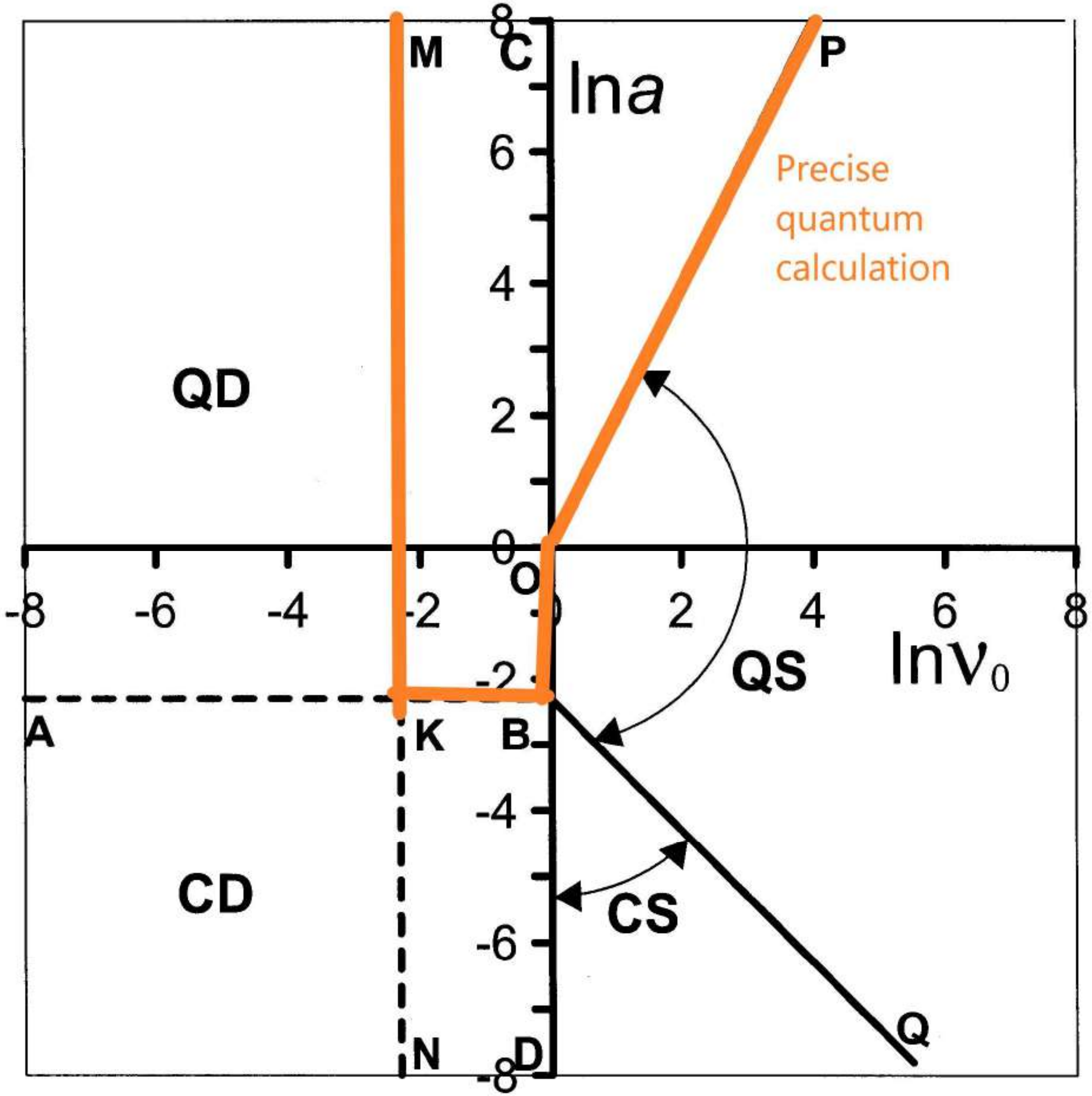


PLANAR



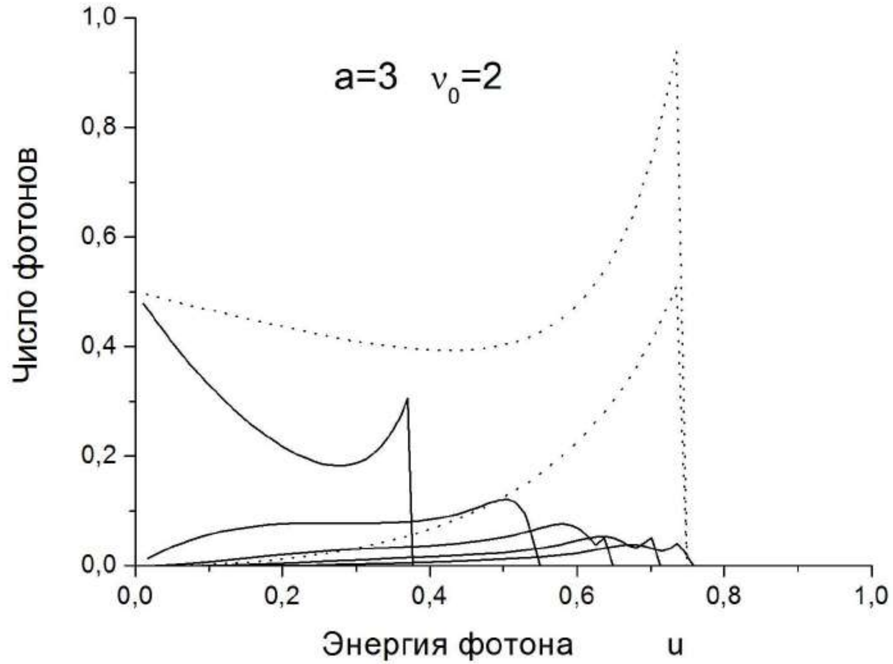
LASER

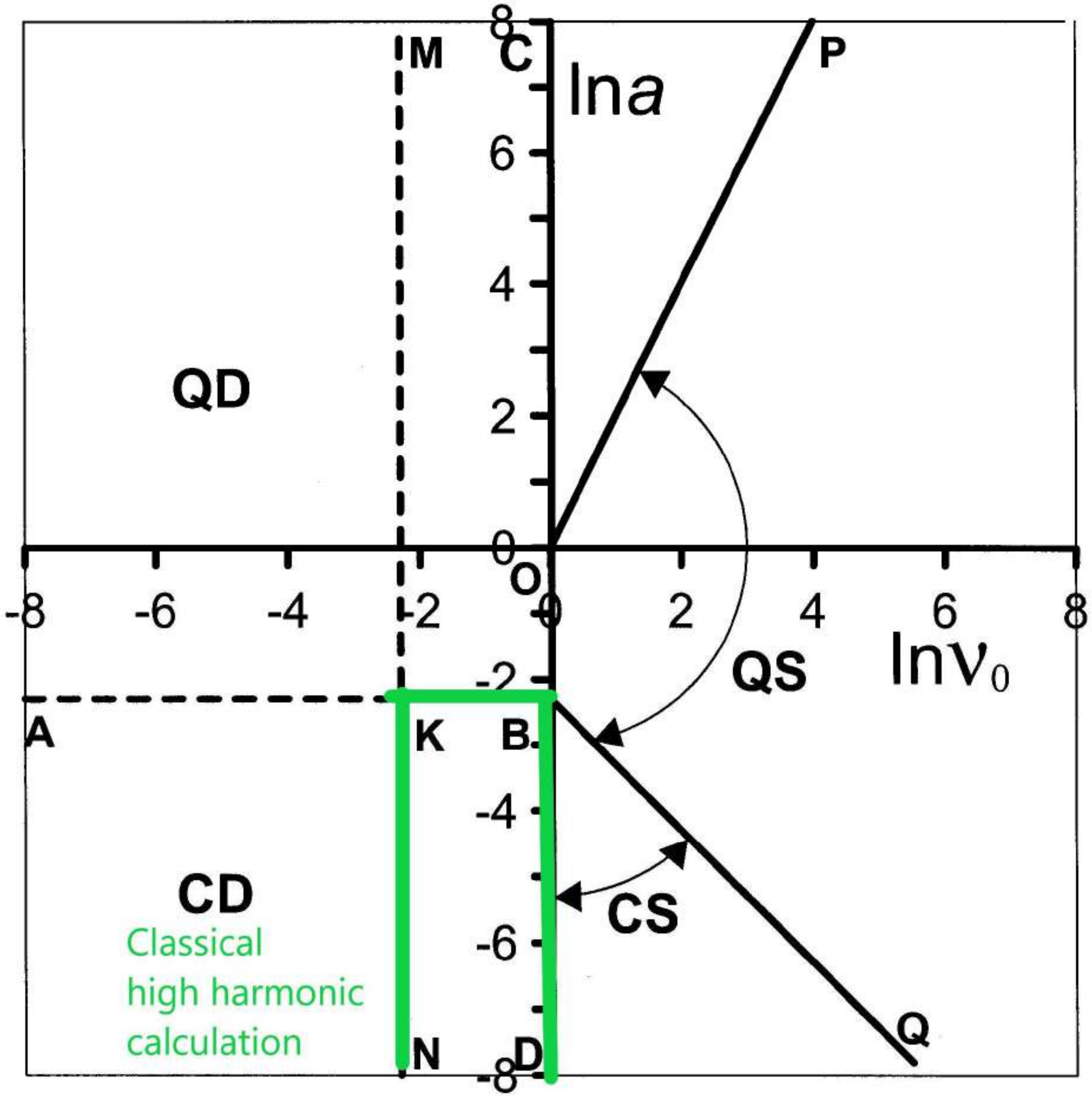




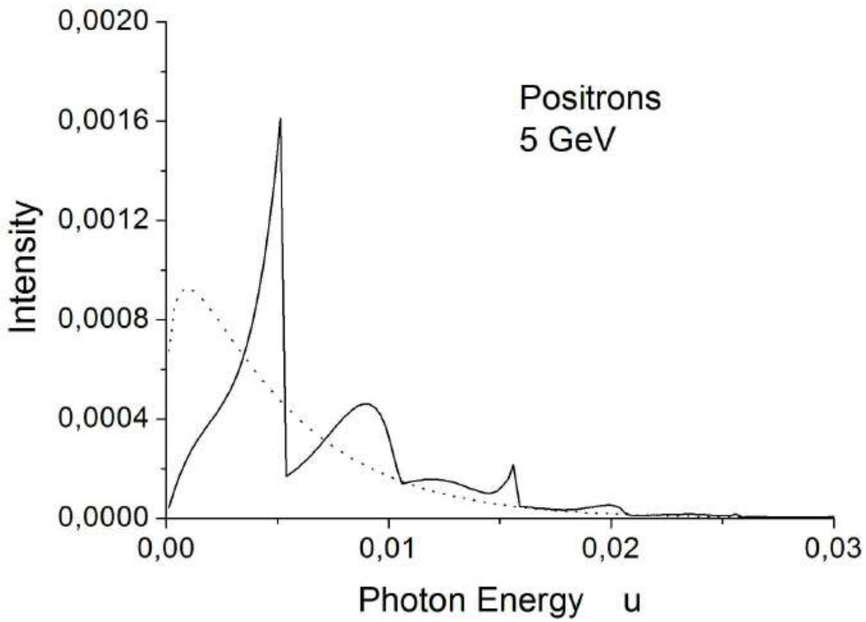
Not possible for channeling?

See previous figure 1

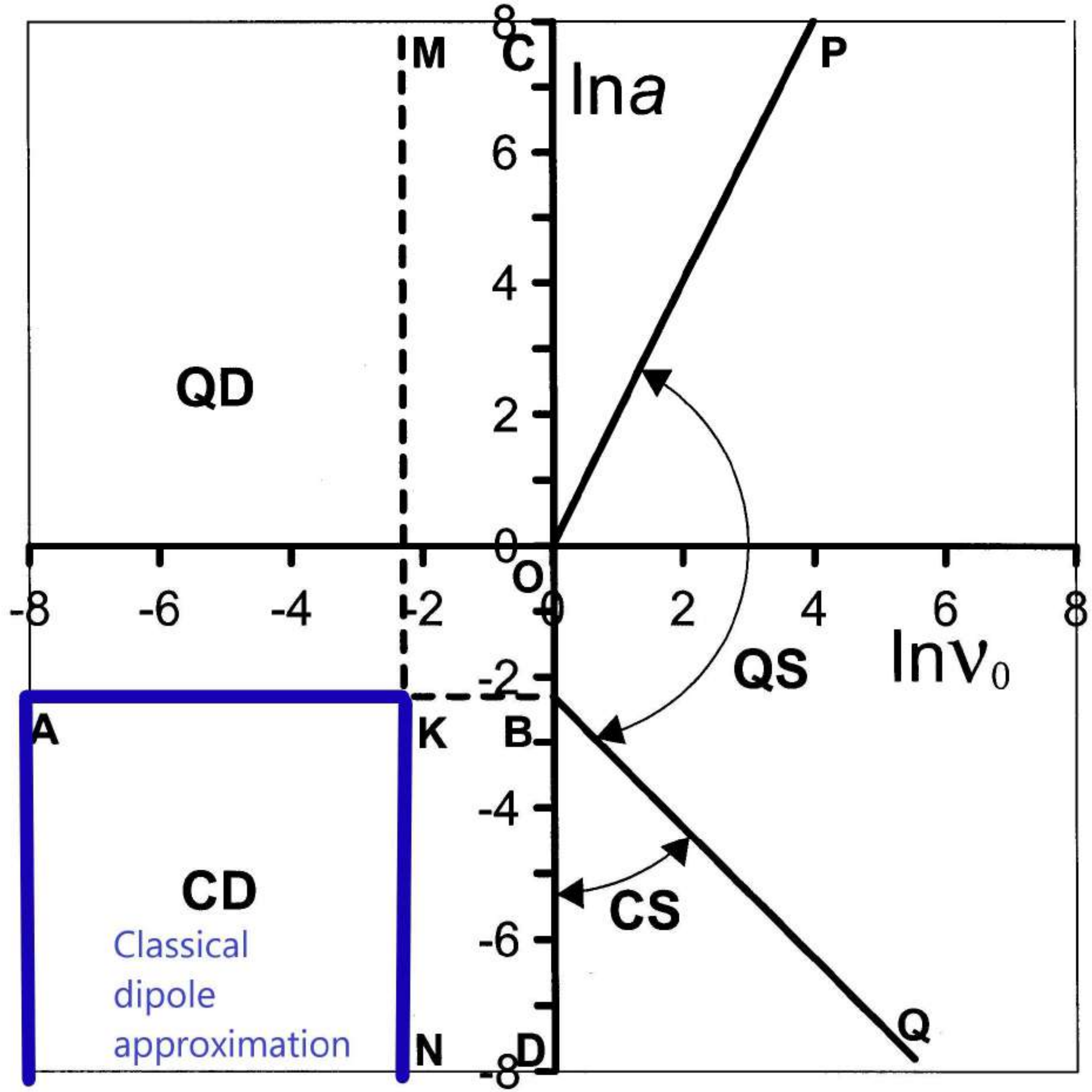




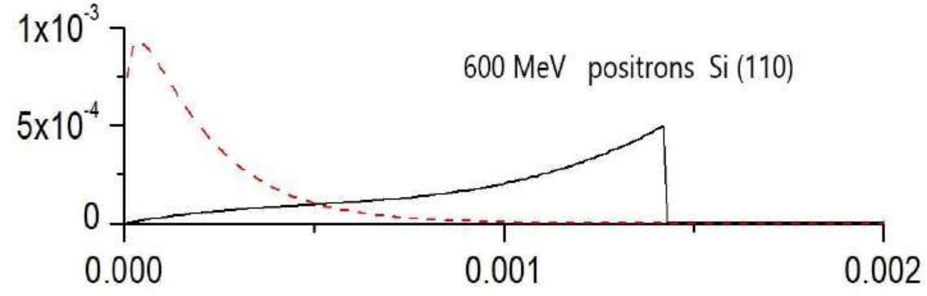
ln*a*
lnν₀
QD
QS
CD
CS
Classical
high harmonic
calculation
M
C
P
O
A
K
B
N
D
Q



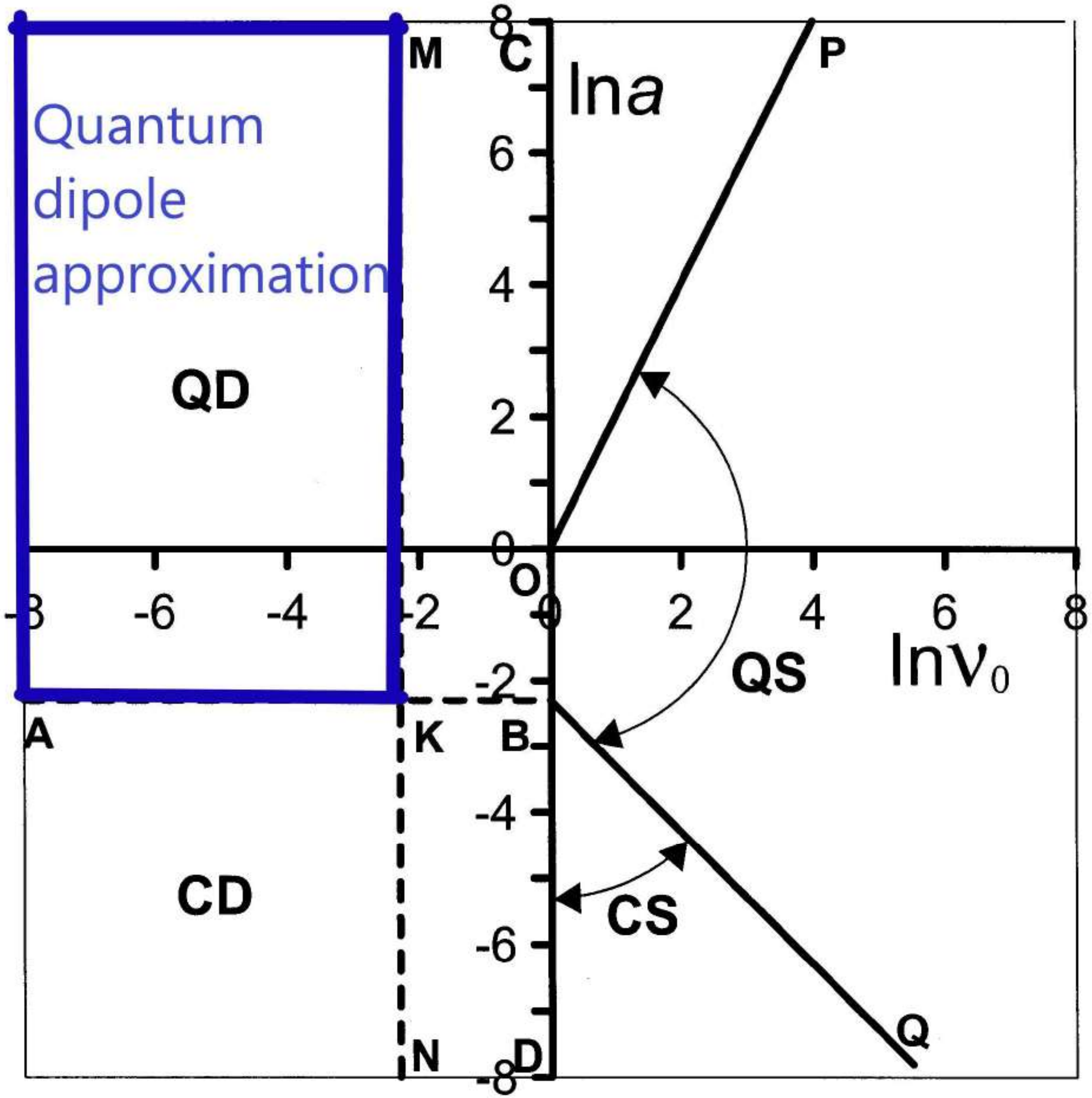
Positrons
5 GeV
Intensity
Photon Energy u



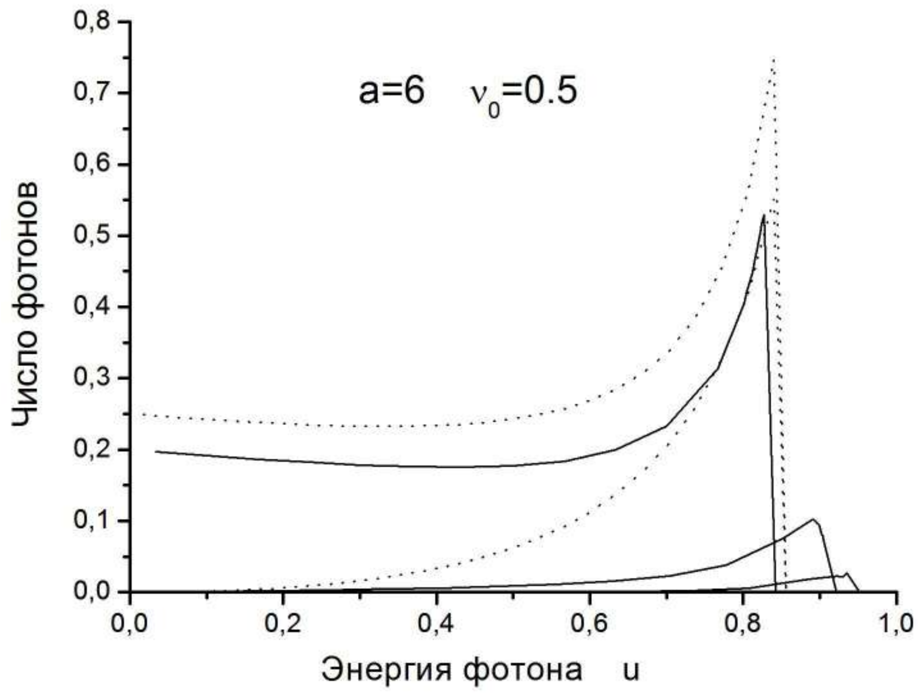
lna
lnν0
M
C
P
QD
QS
O
A
K
B
CD
Classical
dipole
approximation
CS
N
D
Q
8
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6
8

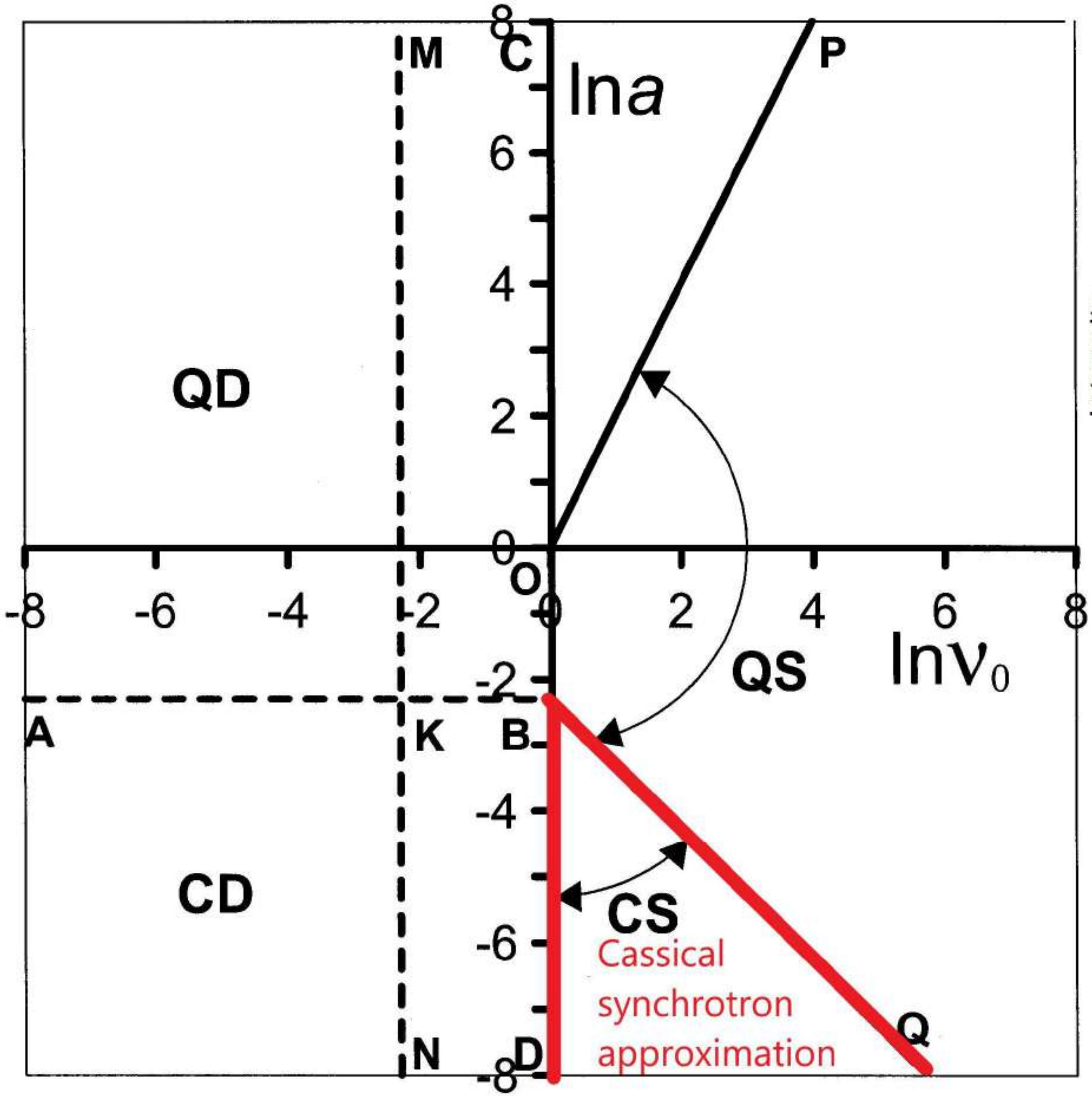


600 MeV positrons Si (110)
1x10-3
5x10-4
0
0.000
0.001
0.002

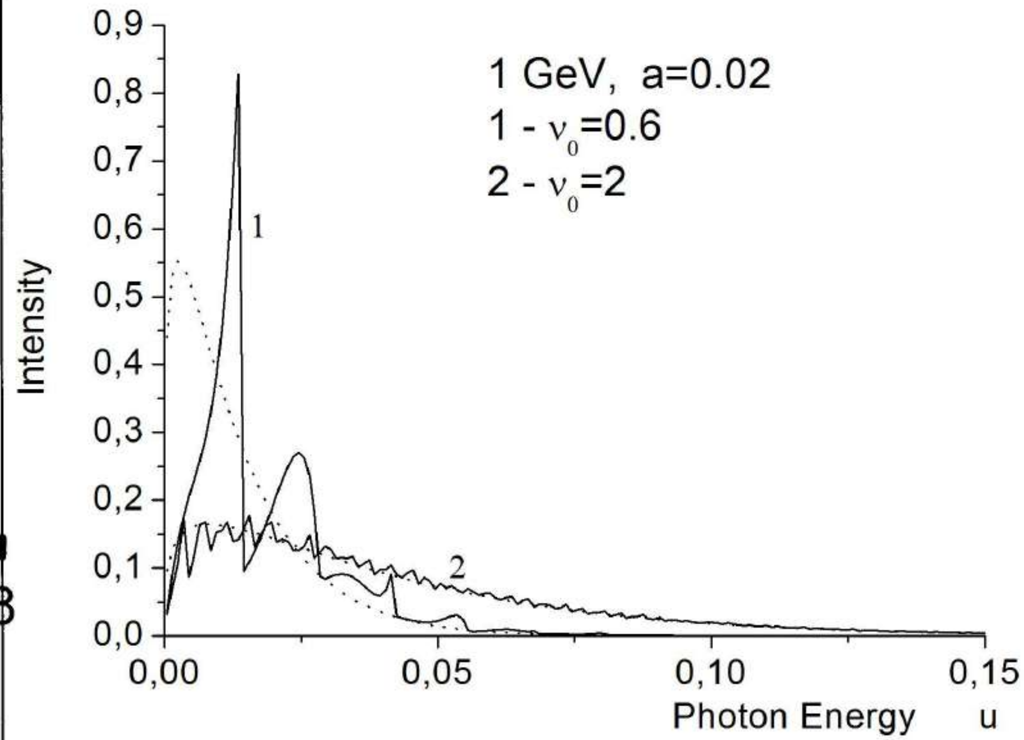


Not possible for channeling



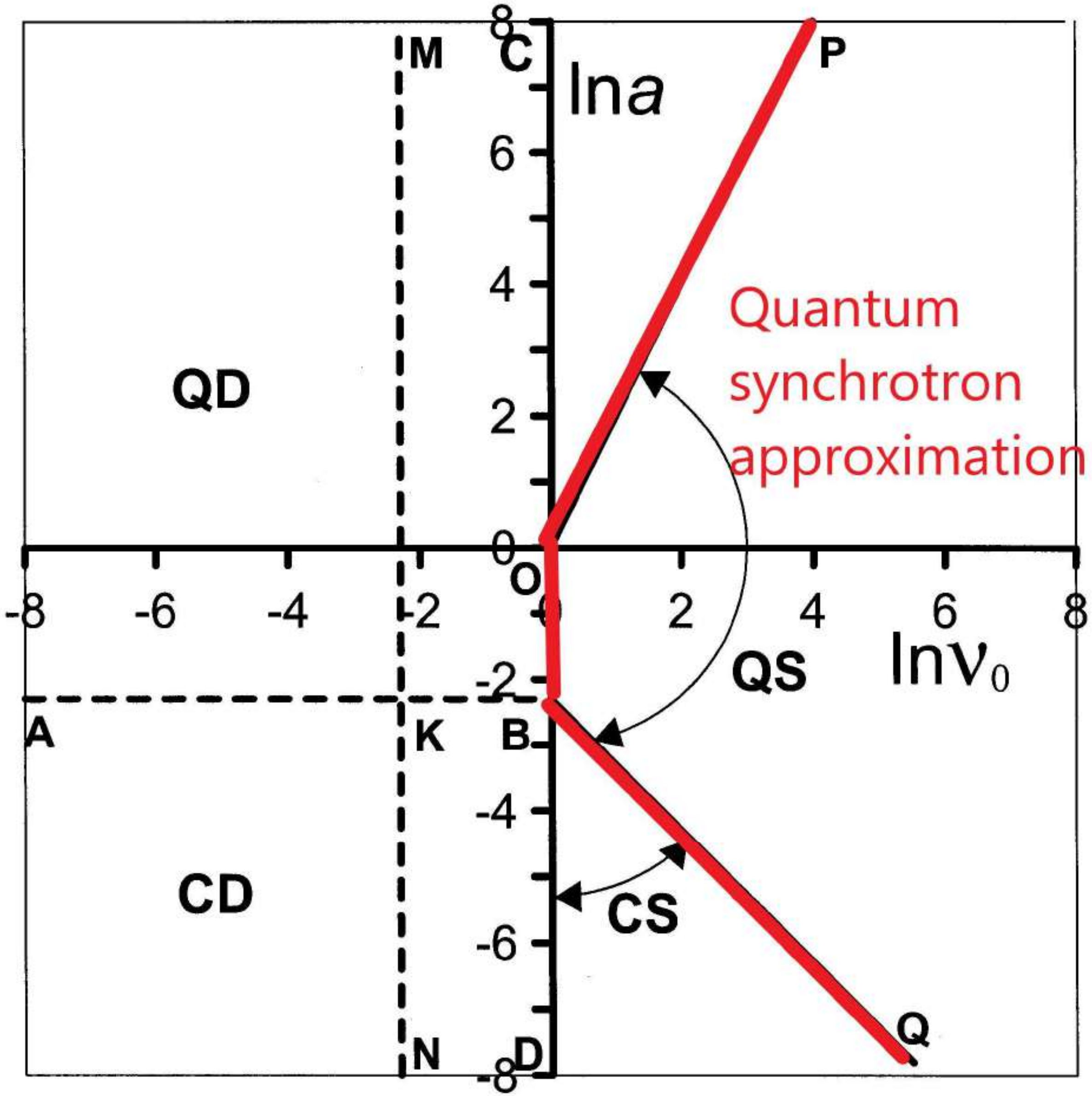


M
C
P
ln*a*
QD
CD
QS
CS
O
A
K
B
N
D
Q
lnν_0
Cassical
synchrotron
approximation



1 GeV, a=0.02
1 - ν_0=0.6
2 - ν_0=2
Intensity
Photon Energy u

See curve 2



M
C
P
ln a
Quantum
synchrotron
approximation
QD
O
QS
ln ν0
A
K
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CD
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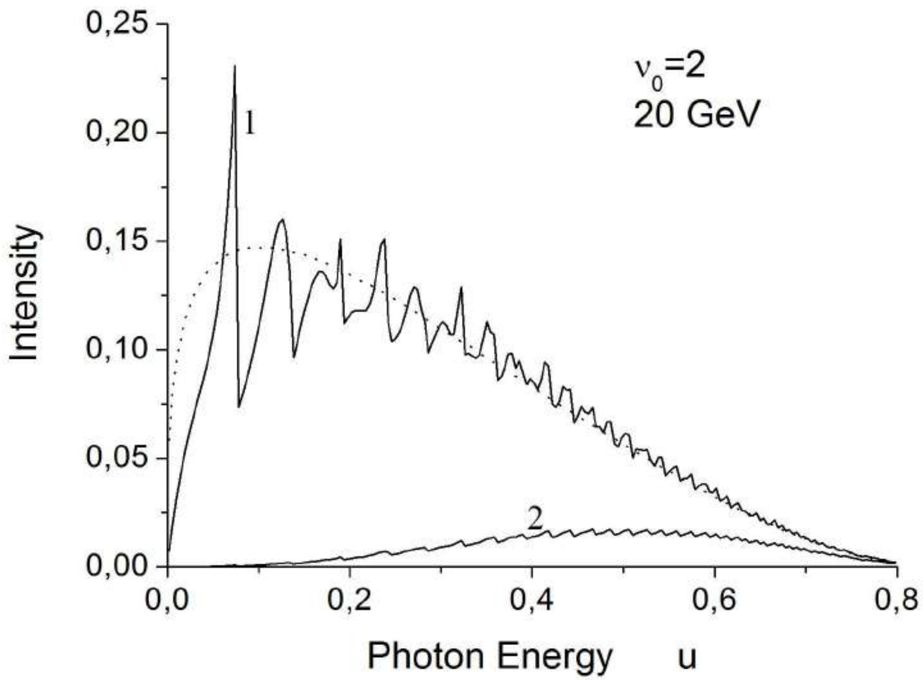
D
Q



ν0=2
20 GeV
Intensity
Photon Energy u
1
2

Выводы

1. Параметры излучения для кристаллов и тераваттных лазеров почти совпадают для энергий до нескольких сот МэВ.
2. Спектры излучения для кристаллов и лазеров можно описать одним и тем же выражением, зависящим только от двух инвариантов a и D. В случае кристаллов область значений этих инвариантов сильно ограниченна, так как они лежат на одной линии.
3. С ростом энергии пучка применимость синхротронного приближения для каналирования становится лучше, а для лазеров, наоборот.
4. При одной и той же энергии пучка излучение в поле лазеров смещено в область более жёстких частот.